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Almost unbiased estimators for population mean in the presence of non-response and measurement error

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Abstract

In this paper, we have proposed three classes of almost unbiased estimators for population mean under simultaneous presence of measurement and non-response error. Asymptotic properties such as Bias and MSE for the proposed classes of estimators are obtained. Numerical illustration in support of theoretical results is also given on two real data sets and a simulated data set using R. Results indicate the superiority of proposed classes of estimators over existing estimators.

Subject Classification: *62D05.*

Keywords: *Measurement error, Non-response, Bias, Precision, Almost unbiased.*

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1. Introduction

It is suitably ascertained that proper use of auxiliary or prior information yields a more precise estimate of population parameter(s). Ratio, Product and Regression methods of estimation are to be noted in this context. Keeping above fact in mind many authors have suggested estimators for population parameter(s) including [1], [2] and [3] etc. In Statistical analysis it is usually presumed that that all the observations are measured correctly. However, in practical situations this assumption is somewhat violated and the true value of the variable cannot be recorded. In such cases there is always some associated error with the variable that leads to deviation between true and observed values of the variable. Such deviations are termed as measurement error. Several authors including [4], [5], [6], [7] and [8] considered estimation in the presence of measurement error. Another type of non-sampling error that generally creeps in sampling surveys includes non-response error that is unavoidable. [9] were the first to deal with the problem of non-response by using technique of subsampling from non-respondents. Non-response may generally arise due to non-availability of respondents, refusal to respond, presence of hard core respondants or due to non-understanding of the particular question. The estimation procedures under non-response are considered in literature by many authors for instance [10] [11],[12] etc. In Survey Sampling, situation may arise that both of these errors occur simultaneously. Simultaneous estimation of measurement and non response error for finite population mean is considered by authors namely [13], [14], and [15].Recently,[16] and [17] defined estimation of finite population mean under both measurement error and non-response error.

1.1 *Some Existing Estimators in Literature*

Let Y and X be study and auxiliary variables of population of interest with units $\Omega(U_1, U_2,\ldots,U_N)$ from which sample of size n is selected using SRSWOR scheme. Consider two non-overlapping subgroups from Ω in which units $Y_{(1)i}; i = 1, 2, ..., N_1$ from N is considered as responding group G_R and units $Y_{(2)i}$, i = 1, 2,…, N_2 as non-responding group G_{NR} such that $\sum_{j=1}^{2} N_j = N$. Let n_1 and n_2 be sample sizes from G_R with units y_{1i}, i = 1, 2,…, n_1 and G_{NR} respectively, $\forall\ \sum_{j=1}^{2} n_j = n$. Assuming a subsample of size h_2 with units y_{h2i}, i = 1, 2,…, h_2 from n_2 is re-interviewed and responded $\forall\ \lambda = n_2 / h_2,\ (\lambda > 1)$. Let x_i^*, y_i^* be the measured and X_i^*, Y_i^* actual values of X, Y respectively from Ω, then the measurement error is given by;

$$e^*_{yi} = y^*_i - Y^*_i,\ e^*_{xi} = x^*_i - X^*_i,\ \forall\ y^*_i = Y^*_i + e^*_{yi},\ x^*_i = X^*_i + e^*_{xi} \tag{1.1}$$

where e^*_{yi} and e^*_{xi} are uncorrelated random variables with zero means and constant variances.

1.2 *Notations:*

$S^2_Y = (N-1)^{-1}\sum_i^N (Y_i - \bar{Y})^2$ is population variance of Y.

$S^2_{Y(2)} = (N_2-1)^{-1}\sum_i^{N_2} (Y_{(2)i} - \bar{Y}_2)^2$ is population variance of Y_{2i}.

$S^2_X = (N-1)^{-1}\sum_i^N (X_i - \bar{X})^2$ is population variance of X.

$S^2_{X(2)} = (N_2-1)^{-1}\sum_i^{N_2} (X_{(2)i} - \bar{X}_2)^2$ is population variance of X_{2i}.

$S_{YX} = (N-1)^{-1}\sum_{i=1}^N (Y_i - \bar{Y})(X_i - \bar{X})$ is population covariance of Y and X.

$S_{YX(2)} = (N_2-1)^{-1}\sum_{i=1}^{N_2} (Y_{(2)i} - \bar{Y}_2)(X_{(2)i} - \bar{X}_2)$ is population covariance of Y_2 and X_2.

$S^2_U = (N-1)^{-1}\sum_{i=1}^N e^{*2}_{yi}$ is the population variance of ME e^*_y

$S^2_V = (N-1)^{-1}\sum_{i=1}^N e^{*2}_{xi}$ is the population variance of ME e^*_x

$S^2_{U(2)} = (N_2-1)^{-1}\sum_{i=1}^{N_2} e^2_{y(2)i}$ is population variance of ME $e^*_{y(2)}$ from G_{NR}.

$S^2_{V(2)} = (N_2-1)^{-1}\sum_{i=1}^{N_2} e^2_{x(2)i}$ is population variance of ME $e^*_{x(2)}$ from G_{NR}.

[9], estimator when study variable is characterized by non-response and measurement error is given as;

$$\bar{y}^*_e = n_1 n^{-1}\bar{y}_{1(e)} + n_2 n^{-1}\bar{y}_{h_2(e)} \tag{1.2}$$

where $\bar{y}_{1(e)} = n_1^{-1}\sum_{i=1}^{n_1} y^*_{1i},\ \bar{y}_{h2(e)} = h_2^{-1}\sum_{i=1}^{h_2} y^*_{h2i}$

$$Bias(\bar{y}^*_e) = 0 \tag{1.3}$$

$$MSE(\bar{y}^*_e) = k_1(S^2_Y + S^2_U) + k_2(S^2_{Y(2)} + S^2_{U(2)}) = F_0,\ (say) \tag{1.4}$$

Though $Bias(\bar{y}^*_e) = 0$, however, its precision does not only depend on the sample size but adequacy of information contained in the sample. If the sample units is skewed or higher numbers of non-response is experienced, the precision of $\bar{y}^*_e$ will be greatly affected. To overcome this, information on auxiliary variables are used.

[18], ratio estimator in the presence of non-response and measurement error can be defined as;

$$\tau_1 = \bar{y}_e^* \bar{X} / \bar{x}_e^* \quad (1.5)$$

$$Bias(\tau_1) = R^2 F_1 - \bar{X}^{-1} F_2 \quad (1.6)$$

$$MSE(\tau_1) = F_0 + R^2 F_1 - 2RF_2 \quad (1.7)$$

where,

$$\bar{x}_e^* = n_1 n^{-1} \bar{x}_{1(e)} + n_2 n^{-1} \bar{x}_{h_2(e)}$$

$$F_1 = k_1 (S_X^2 + S_V^2) + k_2 (S_{X(2)}^2 + S_{V(2)}^2),$$

$$F_2 = k_1 S_{YX} + k_2 S_{YX(2)},$$

$$R = \bar{Y} / \bar{X}$$

[19], gave product estimator as an alternative to ratio estimator when $\rho_{YX} < 0$. This estimator can be defined under non-response and measurement error as;

$$\tau_2 = \bar{y}_e^* \bar{x}_e^* / \bar{X} \quad (1.8)$$

$$Bias(\tau_2) = \bar{X}^{-1} F_2 \quad (1.9)$$

$$MSE(\tau_2) = F_0 + R^2 F_1 + \ RF_2$$

[20], suggested ratio and product exponential-type estimators. These estimators when both study and auxiliary variables are characterized by non-response and measurement error are defined as;

$$\tau_3 = \bar{y}_e^* \exp\left(\frac{\bar{X} - \bar{x}_e^*}{\bar{X} + \bar{x}_e^*} \right) \quad (1.11)$$

$$\tau_4 = \bar{y}_e^* \exp\left(\frac{\bar{x}_e^* - \bar{X}}{\bar{x}_e^* + \bar{X}} \right) \quad (1.12)$$

$$Bias(\tau_3) = \frac{3}{8} RX^{-1} F_1 - \frac{1}{2} \bar{X}^{-1} F_2 \quad (1.13)$$

$$Bias(\tau_4) = -\frac{1}{8} RX^{-1} F_1 + \frac{1}{2} \bar{X}^{-1} F_2 \quad (1.14)$$

$$MSE(\tau_3) = F_0 + 0.25 R^2 F_1 - RF_2 \quad (1.15)$$

$$MSE(\tau_4) = F_0 + 0.25R^2F_1 + RF_2 \tag{1.16}$$

[21], transformed both sample and population mean of x using known information of x, a_x and b_x in [20] estimators. a_x and b_x are either any positive numbers, coefficients of skewness, kurtosis, variation or standard deviation. The estimators obtained under non-response and measurement error are defined as;

$$\tau_5 = \bar{y}_e^* \exp\left(\frac{\left(a_x\bar{X}+b_x\right)-\left(a_x\bar{x}_e^*+b_x\right)}{\left(a_x\bar{X}+b_x\right)+\left(a_x\bar{x}_e^*+b_x\right)}\right) \tag{1.17}$$

$$\tau_6 = \bar{y}_e^* \exp\left(\frac{\left(a_x\bar{x}_e^*+b_x\right)-\left(a_x\bar{X}+b_x\right)}{\left(a_x\bar{x}_e^*+b_x\right)+\left(a_x\bar{X}+b_x\right)}\right) \tag{1.18}$$

$$Bias(\tau_5) = \frac{3}{8}\bar{Y}\Upsilon^2F_1 - \frac{1}{2}\Upsilon F_2 \tag{1.19}$$

$$Bias(\tau_6) = -\frac{1}{8}\bar{Y}\Upsilon^2F_1 + \frac{1}{2}\Upsilon F_2 \tag{1.20}$$

$$MSE(\tau_5) = F_0 + 0.25\bar{Y}^2\Upsilon^2F_1 - \bar{Y}\Upsilon F_2 \tag{1.21}$$

$$MSE(\tau_6) = F_0 + 0.25\bar{Y}^2\Upsilon^2F_1 + \bar{Y}\Upsilon F_2 \tag{1.22}$$

where $\Upsilon = a_x/(a_x\bar{X}+b_x)$

[22], suggested dual-to [20] estimators using information on the sample yet to be drawn. The suggested estimators when both x and y are characterized by non-response and measurement error are defined as;

$$\tau_7 = \bar{y}_e^* \exp\left(\frac{\bar{x}_{te}^* - \bar{X}}{\bar{x}_{te}^* + \bar{X}}\right) \tag{1.23}$$

$$\tau_8 = \bar{y}_e^* \exp\left(\frac{\bar{X} - \bar{x}_{te}^*}{\bar{X} + \bar{x}_{te}^*}\right) \tag{1.24}$$

$$Bias(\tau_7) = \frac{3}{8}g^2RX^{-1}F_1 - \frac{1}{2}g\bar{X}^{-1}F_2 \tag{1.25}$$

$$Bias(\tau_8) = -\frac{1}{8}g^2RX^{-1}F_1 + \frac{1}{2}g\bar{X}^{-1}F_2 \tag{1.26}$$

$$MSE(\tau_7) = F_0 + 0.25g^2R^2F_1 - gRF_2 \tag{1.27}$$

$$MSE(\tau_8) = F_0 + 0.25g^2R^2F_1 + gRF_2 \tag{1.28}$$

where, $g = n/(N-n)$, $\bar{x}_{te}^* = (1+g)\bar{x}_e^* + g\bar{X}$

[23], linearly combined τ_3 and τ_7 to produce new estimator of population mean. The estimator obtained under non-response and measurement error is defined as;

$$\tau_9 = \bar{y}_e^*\left(m\exp\left(\frac{\bar{X}-\bar{x}_e^*}{\bar{X}+\bar{x}_e^*}\right) + (1-m)\exp\left(\frac{\bar{x}_{te}^*-\bar{X}}{\bar{x}_{te}^*+\bar{X}}\right)\right) \tag{1.29}$$

$$Bias(\tau_9) = \frac{1}{2\bar{X}}gF_2 - \frac{1}{8\bar{X}}g^2RF_1 - m\left(\frac{g-1}{2\bar{X}}F_2 + \frac{3+g^2}{8\bar{X}}RF_1\right) \tag{1.30}$$

$$MSE(\tau_9) = F_0 - F_2^2 \quad F_1$$

τ_9 is at optimum when $m = g/(g-1) - 2F_2/((g-1)RF_1)$.

Estimators defined in Eq. (1.5), Eq. (1.8), Eq. (1.11), Eq. (1.12), Eq. (1.17), Eq. (1.18), Eq.(1.23), Eq.(1.24) and Eq.(1.29) though biased, produce estimates with higher precision especially when x and y are correlated when compared to Eq. (1.12). This implies that they are characterized by either over or under estimation which in turn undermined decisions based on their results.

In this paper, we have proposed three classes of estimators of population mean with higher efficiency and almost bias-free to overcome challenges of over or under estimation.

2. Proposed Estimators and their Properties

Motivated by [24], three classes of almost unbiased estimators of population mean were developed as follow;

$$\mu_y^* = \sum_{j=0}^{2}\omega_j\gamma_j \tag{2.1}$$

$$\Lambda_y^* = \sum_{j=0}^{2}\phi_j\upsilon_j \tag{2.2}$$

$$\Pi_y^* = \sum_{j=0}^{2} \eta_j \pi_j \tag{2.3}$$

Where $\gamma_0 = \upsilon_0 = \pi_0 = \overline{y}_e^*, \gamma_1 = \upsilon_1 = \overline{y}_e^*\left(\frac{a_x\mu_X + b_x}{a_x\overline{x}_e^* + b_x}\right), \pi_1 = \tau_5,$

$\gamma_1 = \overline{y}_e^*\left(\frac{a_x\overline{x}_e^* + b_x}{a_x\mu_X + b_x}\right), \upsilon_2 = \pi_2 = \tau_6, \sum_{j=0}^{2}\omega_j = \sum_{j=0}^{2}\phi_j = \sum_{j=0}^{2}\eta_j = 1.$

Remark 2.1 : Note that $a_x \neq b_x$ and $a_x \neq 0$.

To derive the bias and MSE of the proposed class of estimators, we define the following error terms $\xi_0^* = \overline{y}_e^* - \overline{Y}, \xi_1^* = \overline{x}_e^* - \overline{X}$ such that $\left|\xi_i^*\right| \approx 0, i = 0,1.$ The expectations of ξ_i^* up to $O(n^{-1})$ are given as;

$$\left.\begin{array}{l} E(\xi_0^*) = E(\xi_1^*) = 0,\ E(\xi_0^{*2}) = F_0 \\ E(\xi_1^{*2}) = F_1,\ E(\xi_0^*\xi_1^*) = F_2 \end{array}\right\} \tag{2.4}$$

2.1 *Bias and MSE of estimators* μ_y^*

Expressing Eq. (2.1) in terms of $\xi_i^*, i = 0,1.$ We have

$$\mu_y^* = (\overline{Y} + \xi_0^*)\left(\omega_0 + \omega_1(1 + \Upsilon\xi_1^*)^{-1} + \omega_2(1 + \Upsilon\xi_1^*)\right) \tag{2.5}$$

where $\Upsilon = a_x / (a_x\mu_X + b_x)$

Simplifying Eq. (2.5) to $O(n^{-1})$, we have

$$\mu_y^* - \overline{Y} = \xi_0^* - \Upsilon(\omega_2 - \omega_3)\xi_0^*\xi_1^* - \overline{Y}\Upsilon(\omega_2 - \omega_3)\xi_1^* + \overline{Y}\Upsilon^2\omega_2\xi_1^{*2} \tag{2.6}$$

Taking expectation of Eq. (2.6) and using the results of Eq. (2.4), we get $Bias(\mu_y^*)$ up to $O(n^{-1})$ as;

$$Bias(\mu_y^*) = \overline{Y}\omega_2\Upsilon^2F_1 - \Upsilon(\omega_2 - \omega_3)F_2 \tag{2.7}$$

Squaring Eq. (2.6), taking expectation and applying the results of Eq.(2.4), we get $MSE(\mu_y^*)$ up to $O(n^{-1})$ as;

$$MSE(\mu_y^*) = F_0 + \overline{Y}^2\Upsilon^2\theta^2F_1 - 2\overline{Y}\Upsilon\theta F_2 \tag{2.8}$$

where $\theta = \omega_2 - \omega_3$

$\partial\mu_y^* / \partial\theta = 0$ and solve for θ, we obtain $\theta = F_2 / \overline{Y}\Upsilon F_1 = \theta_{opt}$ and $MSE(\mu_y^*)_{\min}$ is obtained as;

$$MSE(\mu_y^*)_{\min} = F_0 - F_2^2 / F_1 \tag{2.9}$$

To obtain the expression for ω_j, j = 0, 1, 2 such that Bias(μ_y^*) ≈ 0 + $O(n^{-1})$, p = 1, 2,…, we consider expressions $\sum_{j=0}^{2}\omega_j = 1,\ \omega_2 - \omega_3 = \theta_{opt}$ which minimize $MSE(\mu_y^*)$ and $\sum_{j=0}^{2} w_j Bias(\tau_j) = 0.$ The expressions can be expressed in matrix form as

$$\begin{pmatrix} 1 & 1 & 1 \\ \text{üüüü} & & - \\ 0 & B(\tau_1) & B(\tau_2) \end{pmatrix}\begin{pmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{pmatrix} = \begin{pmatrix} 1 \\ \theta_{opt} \\ 0 \end{pmatrix}$$

solving Eq. (2.10), we get

$$\left.\begin{aligned} \omega_1 &= 1 + \theta_{opt} - 2\theta_{opt}^2 \\ \omega_2 &= \theta_{opt}^2 \\ \omega_3 &= -\theta_{opt} + \theta_{opt}^2 \end{aligned}\right\} \tag{2.11}$$

2.2 *Bias and MSE of* Λ_y^*

Expressing Eq. (2.2) in terms of ξ_i^*, i = 0, 1. We have

$$\Lambda_y^* = (\bar{Y} + \xi_0^*)\left(\phi_0 + \phi_1(1 + \Upsilon\xi_1^*)^{-1} + \phi_2 \exp\left(\frac{\Upsilon}{2}\xi_1^*\left(1 + \frac{\Upsilon}{2}\xi_1^*\right)^{-1}\right)\right) \tag{2.12}$$

Simplifying Eq. (2.12) to $O(n^{-1})$, we have

$$\begin{aligned} \Lambda_y^* - \bar{Y} &= \xi_0^* - \Upsilon\left(\phi_2 - \phi_3/2\right)\xi_0^*\xi_1^* - \bar{Y}\Upsilon\left(\phi_2 - \phi_3/2\right)\xi_1^* \\ &\quad + \bar{Y}\Upsilon^2\left(\phi_2 - \phi_3/8\right)\xi_1^{*2} \end{aligned} \tag{2.13}$$

Take expectation of Eq. (2.13) and substituting the results of Eq. (2.4), we get $Bias(\Lambda_y^*)$ up to $O(n^{-1})$ as;

$$Bias(\Lambda_y^*) = \bar{Y}(\phi_2 - \phi_3/8)\Upsilon^2 F_1 - \Upsilon(\phi_2 - \phi_3/2)F_2 \tag{2.14}$$

Squaring Eq. (2.13), taking expectation and applying the results of Eq.(2.4), we get $MSE(\Lambda_y^*)$ up to $O(n^{-1})$ as;

$$MSE(\Lambda_y^*) = F_0 + \bar{Y}^2\Upsilon^2\vartheta^2 F_1 - 2\bar{Y}\Upsilon\vartheta F_2 \tag{2.15}$$

where $\vartheta = \phi_2 - \phi_3/2$

$\partial\Lambda_{yi}^*/\partial\vartheta = 0$ and solve for ϑ, we obtain $\vartheta = F_2/\bar{Y}\Upsilon F_1 = \theta_{opt}$ and $MSE(\Lambda_y^*)_{\min}$ is obtained as;

$$MSE(\Lambda_y^*)_{\min} = F_0 - F_2^2 \quad F_1$$

To obtained the expression for $\omega_j, j = 0,1,2$ such that Bias(Λ_y^*) ≈ 0 + $O(n^{-1})$, p = 1, 2,…, we consider expressions $\sum_{j=0}^{2}\phi_j = 1,\ \phi_2 - \phi_3/2 = \theta_{opt}$ which minimize $MSE(\Lambda_{yi}^*)$ and $\sum_{j=0}^{2}\phi_j Bias(\upsilon_j) = 0.$ The expressions can be expressed in matrix form as

$$\begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & -1/2 \\ 0 & B(\tau_1) & B(\tau_2) \end{pmatrix}\begin{pmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \end{pmatrix} = \begin{pmatrix} 1 \\ \theta_{opt} \\ 0 \end{pmatrix} \tag{2.17}$$

Solving Eq. (2.17), we get

$$\left.\begin{aligned} \phi_1 &= 1 + 3\theta_{opt} - 4\theta_{opt}^2 \\ \phi_2 &= -\theta_{opt}(1 - 4\theta_{opt})/3 \\ \phi_3 &= -8\theta_{opt}(1 - \theta_{opt})/3 \end{aligned}\right\} \tag{2.18}$$

2.3 *Bias and MSE* Π_y^*

Expressing Eq. (2.3) in terms of $\xi_i^*, i = 0,1.$ We have

$$\Pi_y^* = (\bar{Y} + \xi_0^*)\left(\eta_0 + \eta_1 \exp\left(-\frac{\Upsilon\xi_1^*}{2}\left(1 + \frac{\Upsilon\xi_1^*}{2}\right)^{-1}\right) + \eta_2 \exp\left(\frac{\Upsilon\xi_1^*}{2}\left(1 + \frac{\Upsilon\xi_1^*}{2}\right)^{-1}\right)\right) \tag{2.19}$$

Simplifying Eq. (2.19) up to $O(n^{-1})$, we have

$$\Pi_y^* - \bar{Y} = \xi_0^* - \frac{\Upsilon}{2}(\eta_2 - \eta_3)\xi_0^*\xi_1^* - \frac{\Upsilon}{2}\bar{Y}(\eta_2 - \eta_3)\xi_1^* + \frac{3\Upsilon^2}{8}\bar{Y}(\eta_2 - \eta_3)\xi_1^{*2} \tag{2.20}$$

Take expectation of Eq. (2.20) and using the results of Eq. (2.1), we get $Bias(\Pi_y^*)$ up to $O(n^{-1})$ as;

$$Bias(\Pi_y^*) = \frac{\Upsilon^2}{8}\bar{Y}(3\eta_2 - \eta_3)F_1 - \frac{\Upsilon}{2}(\eta_2 - \eta_3)F_2 \tag{2.21}$$

Squaring Eq. (2.20), taking expectation and applying the results of Eq. (2.1), we get $MSE(\Pi_y^*)$ up to $O(n^{-1})$ as;

$$MSE(\Pi_y^*) = F_0 + \frac{\Upsilon^2}{4}\bar{Y}^2\theta^2 F_1 - \Upsilon\bar{Y}\theta F_2 \tag{2.22}$$

$\partial \Pi_y^* / \partial \theta = 0$ and solve for θ, we obtain $\theta = 2F_2 / \bar{Y} \Upsilon F_1 = 2\theta_{opt}$ and $MSE(\Pi_y^*)_{\min}$ is obtained as;

$$MSE\text{üüüü}(\mu_y^*)_{\min} = F_0 - F_2^2 \quad F_1$$

To obtained the expression for $\eta_j, j = 0,1,2$ such that $Bias(\Pi_y^*) \approx 0 + O(n^{-p})$, p = 2, 3,…, we consider expressions $\sum_{j=0}^{2} \eta_j = 1$, $\eta_2 - \eta_3 = 2\theta_{opt}$ which minimize $MSE(\Pi_y^*)$ and $\sum_{j=0}^{2} \eta_j Bias(\pi_j) = 0$. The expressions can be expressed in matrix form as

$$\begin{pmatrix} 1 & 1 & 1 \\ \text{üüüü} & & - \\ 0 & B(\tau_1) & B(\tau_2) \end{pmatrix} \begin{pmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{pmatrix} = \begin{pmatrix} 1 \\ \theta_{opt} \\ 0 \end{pmatrix}$$

Solving Eq. (2.24), we get

$$\left.\begin{aligned} \eta_1 &= 1 + 4\theta_{opt} - 8\theta_{opt}^2 \\ \eta_2 &= -\theta_{opt}(1 - 4\theta_{opt}) \\ \eta_3 &= -\theta_{opt}(3 - 4\theta_{opt}) \end{aligned}\right\} \tag{2.25}$$

2.4 *Asymptotic Optimum Estimators* (*AOEs*) *for* μ_y^*, Λ_y^* *and* Π_y^*

AOEs of μ_y^*, Λ_y^* and Π_y^* depend on the optimum value of θ_{opt} which is a function of unknown parameters $\bar{Y}, F_1, F_2$. For practical purposes, θ_{opt} is replaced by $\hat{\theta}_{opt}$ defined as;

$$\hat{\theta}_{opt} = \hat{F}_2 / \bar{y}_e^* \Upsilon \hat{F}_1 \tag{2.26}$$

where, $\hat{F}_1 = k_1(s_x^2 + s_v^2) + k_2(s_{x(2)}^2 + s_{v(2)}^2)$, $\hat{F}_2 = k_1 s_{yx} + k_2 s_{yx(2)}$,

$$s_x^2 = (n_1 - 1)^{-1} \sum_{i=1}^{n_1} (x_{1i} - \bar{x}_1)^2,\ s_{x(2)}^2 = (h_2 - 1)^{-1} \sum_{i=1}^{h_2} (x_{2i} - \bar{x}_{h_2})^2,$$

$$s_{v(2)}^2 = (h_2 - 1)^{-1} \sum_{i=1}^{n_1} e_{h2i}^{*2},\ s_{yx} = (n_1 - 1)^{-1} \sum_{i=1}^{n_1} (y_{1i} - \bar{y}_1)(x_{1i} - \bar{x}_1),$$

$$s_{yx(2)} = (h_2 - 1)^{-1} \sum_{i=1}^{h_2} (y_{(2)i} - \bar{y}_2)(x_{(2)i} - \bar{x}_2).$$

Using the results from Eq. (2.26), AOEs for μ_y^*, Λ_y^* and Π_y^* denoted by $\hat{\mu}_y^*, \hat{\Lambda}_y^*$ and $\hat{\Pi}_y^*$ respectively are given by;

$$\hat{\mu}_y^* = \sum_{j=0}^{2} \hat{\omega}_j \gamma_j \tag{2.27}$$

$$\hat{\Lambda}_y^* = \sum_{j=0}^{2} \hat{\phi}_j \upsilon_j \tag{2.28}$$

$$\hat{\Pi}_y^* = \sum_{j=0}^{2} \hat{\eta}_j \pi_j \tag{2.29}$$

where, $\hat{\omega}_1 = 1 + \hat{\theta}_{opt} - 2\hat{\theta}_{opt}^2, \hat{\omega}_2 = \hat{\theta}_{opt}^2, \hat{\omega}_3 = -\hat{\theta}_{opt} + \hat{\theta}_{opt}^2, \hat{\phi}_1 = 1 + 3\hat{\theta}_{opt} - 4\hat{\theta}_{opt}^2, \hat{\phi}_2 = -\hat{\theta}_{opt}$ $(1 - 4\hat{\theta}_{opt})/3, \hat{\phi}_3 = -8\hat{\theta}_{opt}(1 - \hat{\theta}_{opt})/3, \hat{\eta}_1 = 1 + 4\hat{\theta}_{opt} - 8\hat{\theta}_{opt}^2, \hat{\eta}_2 = -\hat{\theta}_{opt}(1 - 4\hat{\theta}_{opt}),$ $\hat{\eta}_3 = -\hat{\theta}_{opt}(3 - 4\hat{\theta}_{opt})$.

3. Efficiency Comparisons

This section considers the theoretical comparisons of the suggested estimators with other estimators in the study. Let Ψ denotes either μ_y^*, Λ_y^* and Π_y^*, then

(i) $MSE(\overline{y}_e^*) - MSE(\Psi) > 0$ if $F_2^2 / F_1 > 0$ (3.1)

(ii) $MSE(\tau_1) - MSE(\Psi) > 0$ if $(RF_1 - F_2)^2 > 0$ (3.2)

(iii) $MSE(\tau_2) - MSE(\Psi) > 0$ if $(RF_1 + F_2)^2 > 0$ (3.3)

(iv) $MSE(\tau_3) - MSE(\Psi) > 0$ if $(0.5RF_1 - F_2)^2 > 0$ (3.4)

(v) $MSE(\tau_4) - MSE(\Psi) > 0$ if $(0.5RF_1 + F_2)^2 > 0$ (3.5)

(vi) $MSE(\tau_5) - MSE(\Psi) > 0$ if $(0.5\overline{Y}\Upsilon F_1 - F_2)^2 > 0$ (3.6)

(vii) $MSE(\tau_6) - MSE(\Psi) > 0$ if $(0.5\overline{Y}\Upsilon F_1 + F_2)^2 > 0$ (3.7)

(viii) $MSE(\tau_7) - MSE(\Psi) > 0$ if $(0.5gRF_1 - F_2)^2 > 0$ (3.8)

(xi) $MSE(\tau_4) - MSE(\Psi) > 0$ if $(0.5gRF_1 + F_2)^2 > 0$ (3.9)

4. Empirical Study

To evaluate the efficiency of the suggested estimators with respect to other estimators considered in section 1.1, we use following population;

Population 1: (Source: [24])

$$N = 5000, \bar{Y} = 4.9271;\ \bar{X} = 4.9243;\ S_y^2 = 102.007; S_U = 8.8261; S_x^2 = 101.411,$$
$$S_V = 9.0013; \rho_{YX} = 0.995,\ N_1 = 4500; N_2 = 500;\ n = 100; n_1 = 90; n_2 = 10;$$
$$S_{y(2)}^2 = 99.99174; S_{x2}^2 = 99.8747; S_{U(2)} = 9.1505; S_{V(2)} = 8.756; \rho_{YX2} = 0.9949$$

Population 2: (Source: [24])

$$N = 5000; \bar{Y} = 1.96, \bar{X} = 1.9433;\ S_Y^2 = 25.441; S_U^2 = 6.0404, S_X^2 = 100.228,$$
$$S_V^2 = 6.2244, \rho_{YX} = 0.9808, N_1 = 4000; N_2 = 1000, n = 100, n_1 = 80,\ n_2 = 20;$$
$$S_{y(2)}^2 = 25.877, S_{x(2)}^2 = 25.213, S_{U(2)}^2 = 5.9383, \rho_{YX(2)} = 0.9825, S_{V(2)}^2 = 6.2722$$

Table 1.0

Bias and PRE of $\bar{y}_e^*$, $\tau_i (i = 1, 2, ..., 9)$, μ_y^*, Λ_y^* and Π_y^* for Population 1

Estimators	With Measurement Error				Without Measurement Error			
	h=2		h=5		h=2		h=5	
	Bias	PRE	Bias	PRE	Bias	PRE	Bias	PRE
$\bar{y}_e^*$	0	100	0	100	0	100	0	100
τ_1	0.989749	303.15	1.313387	149.0	0.8926702	546.18	1.19001	180.50
τ_2	0.202276	27.292	0.2049019	30.097	0.2022764	26.258	0.2049019	29.076
τ_3	-0.01041	240.56	0.0131055	170.54	-0.01780	291.87	0.0037153	189.45
τ_4	0.070897	48.013	0.0639321	52.300	0.0733595	46.399	0.0670622	50.749
τ_5	-0.02343	132.78	-0.021183	123.58	-0.024227	137.13	-0.022195	126.58
τ_6	0.029953	76.886	0.0294911	80.349	0.0302183	75.501	0.0298287	79.129
τ_7	-0.00203	101.72	-0.002043	101.36	-0.002029	101.87	-0.002047	101.48
τ_8	0.002051	98.317	0.0020748	98.655	0.0020525	98.174	0.0020761	98.5392
τ_9	0.002051	329.94	0.0020748	178.81	0.0020525	571.23	0.0020761	209.227

Contd...

μ_y^*	2.78e-17	329.94	2.776e-17	178.81	2.776e-17	571.23	0	209.23
Λ_y^*	0	329.94	0	178.81	2.776e-17	571.23	0	209.23
Π_y^*	0	329.94	-2.78e-17	178.81	5.551e-17	571.23	0	209.23

Table 2.0

Bias and PRE of $\overline{y}_e^*$, $\tau_i (i = 1, 2, ..., 9)$, μ_y^*, Λ_y^* and Π_y^* for Population 2

Estimators	With Measurement Error				Without Measurement Error			
	h=2		h=5		h=2		h=5	
	Bias	PRE	Bias	PRE	Bias	PRE	Bias	PRE
$\overline{y}_e^*$	**0**	**100**	**0**	**100**	**0**	**100**	**0**	**100**
τ_1	0.8710999	74.283	1.049966	67.682	0.7962864	84.786	0.9368699	74.554
τ_2	0.2541984	14.922	0.2675041	19.222	0.2541988	12.823	0.2675046	16.843
τ_3	0.0882002	239.72	0.118315	152.96	0.0738863	459.88	0.0966765	195.75
τ_4	0.0553327	32.313	0.0497297	39.743	0.0601042	28.352	0.0569427	35.603
τ_5	-0.014972	124.63	-0.015081	115.73	-0.015349	132.62	-0.01565	120.39
τ_6	0.0187643	80.801	0.0195219	85.702	0.0188904	77.396	0.0197125	83.043
τ_7	-0.002504	102.78	-0.002625	101.91	-0.002510	103.46	-0.002634	102.37
τ_8	0.0025634	97.311	0.0026946	98.112	0.0025659	96.699	0.0026976	97.683
τ_9	0.002563	245.55	0.0026946	158.88	0.00256596	464.81	0.0026976	200.07
μ_y^*	-1.39e-17	245.55	1.39e-17	158.88	0	464.81	2.776e-17	200.07
Λ_y^*	0	245.55	-1.39e-17	158.88	4.163e-17	464.81	2.776e-17	200.07
Π_y^*	0	245.55	-2.78e-17	158.88	0	464.81	4.163e-17	200.07

5. Simulation Study

For the purpose of Simulation study we consider here a bivariate normal population of size N=1000 generated using R programming language. A sample of size n=100 is drawn from it for 10,000 times. The values obtained for the MSE's of estimators without any error, with measurement error, with non-response error and with both measurement and non-response error separately for positive and negative coefficient of correlation are summarized in Table 3.0.

Table 3.0

MSE's of estimators using simulation study.

Estimators	Usual MSE	MSE with Measurement error	MSE with Non-response error	MSE with Measurement error and Non-response error
$\rho_{YX} = 0.9$				
$\bar{y}_e^*$	21.42376	21.83953	36.22803	37.06995
τ_1	6.881803	6.987552	15.97346	22.37459
τ_2	49.44524	50.67296	84.76561	86.04694
τ_3	12.61101	12.79928	21.48777	22.08347
τ_4	33.56017	34.32835	56.90291	57.99378
τ_5	16.51642	16.79446	27.9638	28.67056
τ_6	27.22932	27.81376	46.1372	47.11061
τ_7	12.63707	12.49544	21.34528	21.55881
τ_8	35.1606	36.28736	59.39883	60.86434
τ_9	15.11433	15.51804	30.9213	30.06441
μ_y^*	5.545504	6.548873	13.49362	18.23833
Λ_y^*	5.615851	6.636098	13.95143	19.17609
Π_y^*	5.525462	6.523851	13.36663	17.96166
$\rho_{YX} = -0.9$				
$\bar{y}_e^*$	22.30745	22.95567	37.99039	38.07024
τ_1	48.93643	50.52392	87.10728	89.2312
τ_2	8.628908	8.839788	15.01734	28.56665
τ_3	33.71807	34.76356	58.32556	59.21192
τ_4	14.08811	14.46246	23.9888	29.64649
τ_5	27.96431	28.80251	47.83265	48.27195
τ_6	17.53605	18.0288	29.84615	29.67233
τ_7	32.16561	32.79569	55.3751	55.876
τ_8	14.90672	15.67454	25.20907	27.15659

Contd...

τ_9	51.10579	51.91353	106.5804	113.9965
μ_y^*	5.417634	6.396726	12.81321	24.67648
Λ_y^*	5.387223	6.35713	12.54251	24.97668
Π_y^*	5.410442	6.387537	12.75372	25.39421

6. Conclusion

From the results of Table 1.0 and Table 2.0, it is evident that the proposed classes of estimators μ_y^*, Λ_y^* and Π_y^* are more efficient than estimators considered in Section 1.1 whenever measurement and non–response error occur simultaneously. Also, From Table 1.0 and Table 2.0 it is seen that the proposed classes of estimators Π_y^*, $^*\mu_y^*$ are unbiased whereas Λ_y^* possess negligible bias and is almost unbiased. Results of Simulation study summarized in Table 3.0 also reveal the superiority of proposed classes of almost unbiased estimators over other estimators.

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