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# Current Role of Numerical Analysis in Mathematics

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## ABSTRACT

*Background. Numerical analysis is a branch of Mathematics that deals with devising efficient methods for obtaining numerical solutions to difficult Mathematical problems.*
*Method and material: We conducted this research paper by observing the different types of reviews, as well as conducting and evaluating literature review papers.*
*Result. Numerical analysis is mathematics which has developed efficient methods for obtaining numerical solutions to difficult mathematical problems. Most of the mathematical problems that arise in science and engineering are very difficult and sometimes impossible to solve properly*
*Conclusions. The method of numerical analysis is being used mainly in the fields of mathematics and computer science and is continuously creating and applying algorithms to solve numerical problems of mathematics*

*KEYWORDS: Numerical Analysis, Mathematics, Types, Role, Application etc*

## INTRODUCTION

Numerical analysis is a branch of Mathematics that deals with devising efficient methods for obtaining numerical solutions to difficult Mathematical problems. Most of the Mathematical problems that arise in science and engineering are very hard and sometime impossible to solve exactly. Thus, an approximation to a difficult Mathematical problem is very important to make it easier to solve. Due to the immense development in the computational technology, numerical approximation has become more popular and a modern tool for scientists and engineers. As a result many scientific software are developed (for instance, Matlab, Mathematica, Maple etc.) to handle more difficult problems in an efficient and easy way. This software contains functions that use standard numerical methods, where a user can pass the required parameters and get the results just by a single command without knowing the details of the numerical method. Thus, one may ask why we need to understand numerical methods when such software is at our hands. In fact, there is no need of a deeper knowledge of numerical methods and their analysis in most of the cases in order to use some standard software as an end user.

However, there are at least three reasons to gain a basic understanding of the theoretical background of numerical methods.

1. Learning different numerical methods and their analysis will make a person more familiar with the technique of developing new numerical methods. This is important when the available methods are not enough or not efficient for a specific problem to be solved.

2. In many circumstances, one has more methods for a given problem. Hence, choosing an appropriate method is important for producing an accurate result in lesser time.
3. With a sound background, one can use methods properly (especially when a method has its own limitations and/or disadvantages in some specific cases) and, most importantly, one can understand what is going wrong when results are not as expected.

Numerical Analysis is the Mathematics branch responsible for designing effective ways to find numerical solutions to complex Mathematical problems. Most Mathematical problems from science and engineering are very complex and sometimes cannot be solved directly. Therefore, measuring a complex Mathematical problem is very important to make it easier to solve. Due to the great advances in computational technology, numeracy has become very popular and is a modern tool for scientists and engineers. As a result many software programs are being developed such as Matlab, Mathematic, and Maple etc. the most difficult problems in an effective and simple way. This software contains functions that use standard numeric methods, in which the user can bypass the required parameters and obtain the results in a single command without knowing the numerical details.

**Numerical Method**

Numerical methods are techniques that are used to approximate Mathematical procedures. We need approximations because we either cannot solve the procedure analytically or because the analytical method is intractable (an example is solving a set of a thousand simultaneous linear equations for a thousand unknowns).

**Different Types of Numerical Methods**

The numerical analysts and Mathematicians used have a variety of tools that they use to develop numerical methods for solving Mathematical problems. The most important idea, mentioned earlier, that cuts across all sorts of Mathematical problems is that of changing a given problem with a 'near problem' that can be easily solved. There are other ideas that differ on the type of Mathematical problem solved.

An Introduction to Numerical Methods for Solving Common Division Problems Given Below:

- **Euler method** - The most basic way to solve ODE
- **Clear and vague methods** - Vague methods need to solve the problem in every step
- **The Euler Back Road** - The obvious variation of the Euler method
- **Trapezoidal law** - The direct method of the second system
- **Runge-Kutta Methods** - One of the two main categories of problems of the first value.

**Numerical Methods**

**Newton method**

Some calculations cannot be solved using algebra or other Mathematical methods. For this we need to use numerical methods. Newton's method is one such method and allows us to calculate the solution of f (x) = 0.

**Simpson Law**

The other important ones cannot be assessed in terms of integration rules or basic functions. Simpson's law is a numerical method that calculates the numerical value of a direct combination.

**Trapezoidal law**

A trapezoidal rule is a numerical method that calculates the numerical value of a direct combination. The other important ones cannot be assessed in terms of integration rules or basic functions.

**Numerical Computation**

The term “numerical computations” means to use computers for solving problems involving real numbers. In this process of problem-solving, we can distinguish several more or less distinct phases. The first phase is formulation. While formulating a Mathematical model of a physical situation, scientists should take into account the fact that they expect to solve a problem on a computer. Therefore they will provide for specific objectives, proper input data, adequate checks, and for the type and amount of output.

Once a problem has been formulated, then the numerical methods, together with preliminary error analysis, must be devised for solving the problem. A numerical method that can be used to solve a problem is called an algorithm. An algorithm is a complete and unambiguous set of procedures that are used to find the solution to a Mathematical problem. The selection or construction of appropriate algorithms is done with the help of Numerical Analysis. We have to decide on a specific algorithm or set of algorithms for solving the problem, numerical analysts should also consider all the sources of error that may affect the results. They should consider how much accuracy is required. To estimate the magnitude of the round-off and discretization errors, and determine an appropriate step size or the number of iterations required.

The programmer should transform the suggested algorithm into a set of unambiguous that is followed by step-by-step instructions to the computer. The flow chart is the first step in this procedure. A flow chart is simply a set of procedures that are usually written in logical block form, which the computer will follow. The complexity of the flow will depend upon the complexity of the problem and the amount of detail included. However, it should be possible for someone else other than the programmer to follow the flow of information from the chart. The flow chart is an effective aid to the programmer; they must translate its major functions into a program. And, at the same time, it is an effective means of communication to others who wish to understand what the program does.

**Numerical Computing Characteristics**

**Accuracy:** Every numerical method introduces errors. It may be due to the use of the proper Mathematical process or due to accurate representation and change of numbers on the computer.

**Efficiency:** Another consideration in choosing a numerical method for Mathematical model solution efficiency means the amount of effort required by both people and computers to use the method.

**Numerical instability:** Another problem presented by a numerical method is numerical instability. Errors included in the calculation, from any source, increase in different ways. In some cases, these errors are usually rapid, resulting in catastrophic results.

**Numerical Computing Process**

- Construction of a Mathematical model.
- Construction of an appropriate numerical system.
- Implementation of a solution.
- Verification of the solution.

**Trapezoidal Law**

In Mathematics, trapezoidal law, also known as trapezoid law or trapezium law, is the most important measure of direct equity in Numerical Analysis. Trapezoidal law is a coupling law used to calculate the area under a curve by dividing the curve into a small trapezoid. The combination of all the small trapezoid areas will provide space under the curve. Let's understand the trapezoidal law formula and its evidence using examples in future sections.

**Numerical and Statistical Methods**

Numerical methods, as said above, are techniques to approximate Mathematical procedures. On the other hand, statistics is the study and manipulation of data, including ways to gather, review, analyze, and draw conclusions from the given data. Thus we can say, statistical methods are Mathematical formulas, models, and techniques that are used in the statistical analysis of raw research data. The application of statistical methods extracts information from research data and provides different methods to assess the robustness of research outputs. Some common statistical tools and procedures are given below:

- Descriptive
- Mean (average)
- Variance
- Skewness
- Kurtosis
- Inferential
- Linear regression analysis
- Analysis of variance
- Null hypothesis testing

**Introduction to Finite Element Method**

The various laws of physics related to space and time-dependent problems are usually expressed in terms of partial differential equations (PDEs). If we have the vast majority of geometries and problems, these PDEs cannot be solved using analytical methods. Instead of that, we have created an approximation of the equations, typically based upon different types of discretizations. These discretization methods approximate the PDEs with numerical model equations, which can be solved using numerical methods. Thus, the solution to the numerical model equations is, in turn, an approximation of the real solution to the PDEs.

The finite element method is used to compute such approximations.

The finite element method is a numerical technique that is used for solving problems that are described by partial differential equations or can be formulated as functional minimization. A domain of interest is represented by the assembly of finite elements. Approximating functions in finite elements are determined in terms of nodal values of a physical field. A continuous physical problem is transformed into a discretized finite element problem with the help of unknown nodal values. For a linear problem, a system of linear algebraic equations must be solved. We can recover values inside finite elements using the nodal values.

**Two Features of the Fem are mentioned below:**

Piecewise approximation of physical fields on finite elements provides good precision even with simple approximating functions (i.e. increasing the number of elements we can achieve any precision).

Locality of approximation leads to sparse equation systems that are mainly used for a discretized problem. With the help of this, we can solve problems with a very large number of nodal unknowns.

**Typical Classes of Engineering Problems that can be Solved Using Fem is**:

- Structural mechanics
- Heat transfer
- Electromagnetic
- Diffusion
- Vibration

**Finite Element Method MATLAB**

Finite element analysis is a computational method for analyzing the behaviour of physical products under loads and boundary conditions. A typical FEA workflow in MATLAB includes

- Importing or creating geometry.
- Generating mesh.
- Defining physics of the problem with the help of load, boundary and initial conditions.
- Solving and visualizing results.

The design of experiments or optimization techniques can be used along with FEA to perform trade-off studies or to design an optimal product for specific applications.

MATLAB is Very Useful Software and is Very Easy to Apply Finite Element Analysis Using MATLAB. It Helps Us in Applying Fem in Several Ways:

Partial differential equations (PDEs) can be solved using the inbuilt Partial Differential Equation Toolbox.

In MATLAB, with the help of Statistics and Machine Learning Toolbox, we can apply the design of experiments and other statistics and machine learning techniques with finite element analysis.

Also, the optimization techniques can be applied to FEM simulations to come up with an optimum design with Optimization Toolbox.

Parallel Computing Toolbox speeds up the analysis by distributing multiple Finite element analysis simulations to run in parallel.

**Application of numerical analysis**

The field of numerical analysis includes many sub-disciplines. Some of the major ones are:

**Computing values of functions**

One of the simplest problems is the evaluation of a function at a given point. The most straightforward approach, of just plugging in the number in the formula is sometimes not very efficient. For polynomials, a better approach is using the Horner scheme, since it reduces the necessary number of multiplications and additions. Generally, it is important to estimate and control round-off errors arising from the use of floating point arithmetic.

**Interpolation, extrapolation, and regression**

Interpolation solves the following problem: given the value of some unknown function at a number of points, what value does that function have at some other point between the given points?

Extrapolation is very similar to interpolation, except that now the value of the unknown function at a point which is outside the given points must be found.

Regression is also similar, but it takes into account that the data is imprecise. Given some points, and a measurement of the value of some function at these points (with an error), the unknown function can be found. The least squares-method is one way to achieve this.

**Solving equations and systems of equations**

Another fundamental problem is computing the solution of some given equation. Two cases are commonly distinguished, depending on whether the equation is linear or not. For instance, the equation is linear while is not. Much effort has been put in the development of methods for solving systems of linear equations.

Standard direct methods, i.e., methods that use some matrix decompositionare Gaussianelimination, LUdecomposition, Choleskydecomposition for symetric (or hermitian) and positive-definite matrix, and QR decomposition for non-square matrices. Iterative methods such as the Jacobi method, Gauss–Seidel method, successive over-relaxation and conjugate gradient method are usually preferred for large systems. General iterative methods can be developed using a matrix splitting.

Root-finding algorithms are used to solve nonlinear equations (they are so named since a root of a function is an argument for which the function yields zero). If the function is differentiable and the derivative is known, then Newton's method is a popular choice. Linearization is another technique for solving nonlinear equations.

**Solving eigenvalue or singular value problems**

Several important problems can be phrased in terms of eigenvalue decompositions or singular value decompositions. For instance, the spectral image compression algorithm is based on the singular value decomposition. The corresponding tool in statistics is called principal component analysis.

**Optimization**

**Main article: Mathematical optimization**

Optimization problems ask for the point at which a given function is maximized (or minimized). Often, the point also has to satisfy some constraints.

The field of optimization is further split in several subfields, depending on the form of the objective function and the constraint. For instance, linear programming deals with the case that both the objective function and the constraints are linear. A famous method in linear programming is the simple method.

The method of Lagrange multipliers can be used to reduce optimization problems with constraints to unconstrained optimization problems.

**Evaluating integrals**

**Main article: Numerical integration**

Numerical integration, in some instances also known as numerical quadrature, asks for the value of a definite integral. Popular methods use one of the Newton–Cotes formulas (like the midpoint rule or Simpson's rule) or Gaussian quadrature. These methods rely on a "divide and conquer" strategy, whereby an integral on a relatively large set is broken down into integrals on smaller sets. In higher dimensions, where these methods become prohibitively expensive in terms of computational effort, one may use Monte Carlo or quasi-Monte Carlo methods (see Monte Carlo integration), or, in modestly large dimensions, the method of sparse grids.

**Differential equations**

Main articles: Numerical ordinary differential equations and Numerical partial differential equations

Numerical analysis is also concerned with computing (in an approximate way) the solution of differential equations, both ordinary differential equations and partial differential equations.

Partial differential equations are solved by first discretizing the equation, bringing it into a finite-dimensional subspace. This can be done by a finite element method, a finite difference method or (particularly in engineering) a finite volume method. The theoretical justification of these methods often involves theorems from functional analysis. This reduces the problem to the solution of an algebraic equation.

**Method and material**: We conducted this research paper by observing the different types of reviews, as well as conducting and evaluating literature review papers.

**Result.**

Numerical analysis is mathematics which has developed efficient methods for obtaining numerical solutions to difficult mathematical problems. Most of the mathematical problems that arise in science and engineering are very difficult and sometimes impossible to solve properly. Thus, an approximation is very important to make solving a difficult mathematical problem easier. Due to extreme developments in computational technology, numerical approximation has become more popular and has become a modern tool for scientists and engineers. As a result many scientific software have been developed (for example, Matlab, Mathematic, Maple etc.) to handle more difficult problems in an efficient and easy manner. One can get the result by just one command without knowing the details of numerical method. Thus, one may ask why we need to understand numerical methods when such software is in our hands. In fact, to use some standard software as an end user, there is no need for in-depth knowledge of numerical methods and their analysis in most of the cases.

## Future Scope & Conclusion

The method of numerical analysis is being used mainly in the fields of mathematics and computer science and is continuously creating and applying algorithms to solve numerical problems of mathematics. These types of problems usually arise from real-world applications of algebra, geometry, and calculus, and also involve variables that are constantly changing. These problems occur throughout the natural sciences, social sciences, engineering, medicine and business. The introduction of numerical analysis, the increase in power and availability of digital computers during the past half century has led to the increasing use of realistic mathematical models in science and engineering. Here in the future we will learn more about numerical method and numerical methods analysis.


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